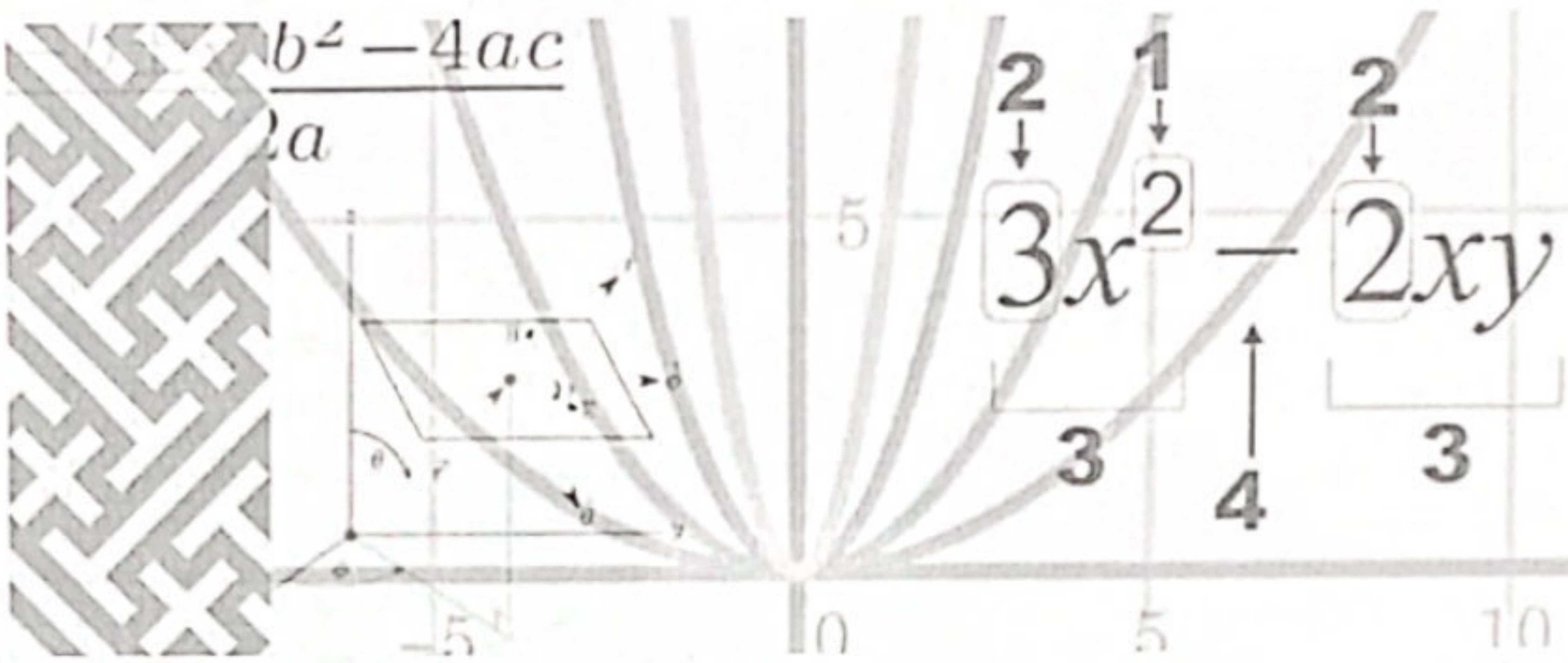


PRE-CALCULUS

SUMMER PACKET

NAME:

PERIOD:

Welcome to Pre-Calculus

Algebra 1 requires students to think, reason, and communicate mathematically. The skills learned during the Algebra 1, Geometry, and Algebra 2 curriculum will be used as a foundation in Pre- Calculus.

Directions:

- The summer packet contains material learned during the previous math curriculum. Because these lessons are pre-requisites for Algebra Pre-Calculus then I expect students to get master on them. Pre-Calculus curriculum does not include these lessons.

- Students MUST show their work for each problem of this review packet. Each problem should be worked through to its entirety, and correctly; not just attempted.

- The packet will be student's the first grade for the new school year.

- Each student should be prepared to have the summer packet completed and ready to checked during the first week of school.

- Over the course of the first two weeks of the beginning of the school year, the packet will be reviewed, and an assessment will be given as the first test grade of the new school year.

- Do not wait until last minute to do it, remember that you will be tested on these lessons.

- Organize your time wisely. For example, you can do a lesson per week. Then you will have plenty of time to finish before the new school year starts.

Do not answer questions 21 and 22 and questions from 70-75. We will solve them in class.

Have a blast and bless summer! 

Pre-Calculus Summer Packet

Radicals:

Rules for Radicals

Quotient Rule

$$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$$

$$\sqrt[3]{\frac{27}{x^6}} = \frac{\sqrt[3]{27}}{\sqrt[3]{x^6}} = \frac{3}{x^{\frac{6}{3}}} = \frac{3}{x^2}$$

Product Rule

$$\sqrt[n]{x \cdot y} = \sqrt[n]{x} \cdot \sqrt[n]{y}$$

$$\sqrt{3x^2} = \sqrt{3}\sqrt{x^2} = x\sqrt{3}$$

1. Write the following expression in exponential form.

$$\sqrt[7]{y}$$

All this problem is asking us to do is basically use the definition of the radical notation and write this in exponential form instead of radical form.

$$\boxed{y^{\frac{1}{7}}}$$

2. $\sqrt[3]{x^2}$

3. $\sqrt[6]{ab}$

4. $\sqrt{w^2v^3}$

5. Evaluate : $\sqrt[4]{81}$

All we need to do here is to convert this to exponential form and then recall that we learned how to evaluate the exponential form in the Rational Exponent section.

$$\sqrt[4]{81} = 81^{\frac{1}{4}} = \boxed{3} \qquad \text{because} \qquad 3^4 = 81$$

6. $\sqrt[3]{-512}$

7. $\sqrt[3]{1000}$

8. Simplify the following expression. Assume that x is positive.

$$\sqrt[3]{x^8}$$

Recall that by simplify we mean we want to put the expression in simplified radical form (which we defined in the notes for this section).

To do this for this expression we'll need to write the radicand as,

$$x^8 = x^6x^2 = \left(x^2\right)^3 x^2$$

Now that we've gotten the radicand rewritten it's easy to deal with the radical and get the expression in simplified radical form.

$$\sqrt[3]{x^8} = \sqrt[3]{\left(x^2\right)^3 x^2} = \sqrt[3]{\left(x^2\right)^3}\sqrt[3]{x^2} = \boxed{x^2\sqrt[3]{x^2}}$$

9. $\sqrt{8y^3}$

10. $\sqrt[4]{x^7y^{20}z^{11}}$

11. $\sqrt[3]{54x^6y^7z^2}$

12. $\sqrt[4]{4x^3y}\ \sqrt[4]{8x^2y^3z^5}$

13. Multiply the following expression. Assume that x is positive.

$$\sqrt{x}\left(4-3\sqrt{x}\right)$$

All we need to do here is do the multiplication so here is that.

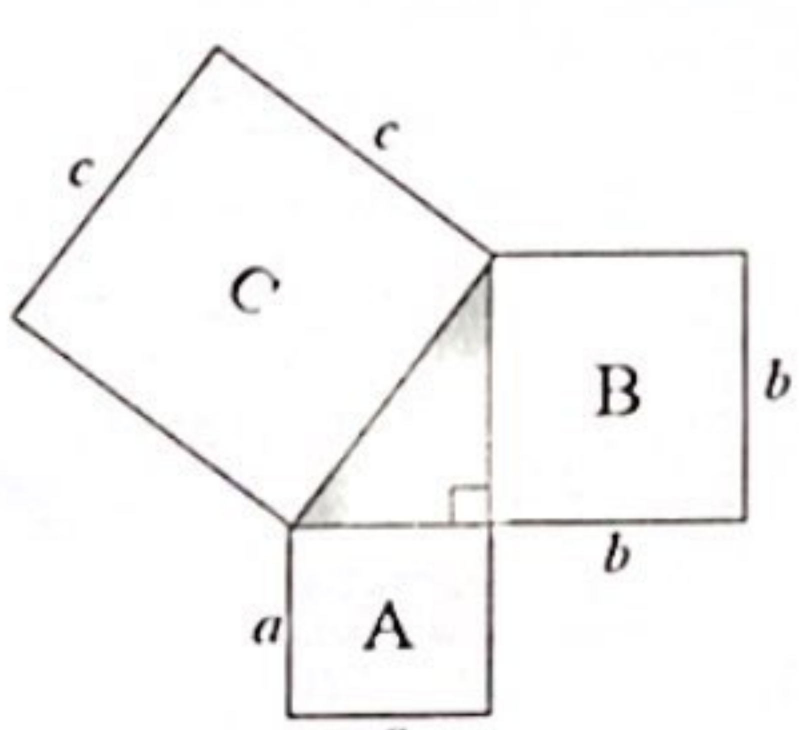
$$\sqrt{x}\left(4-3\sqrt{x}\right)=4\sqrt{x}-3\sqrt{x}\left(\sqrt{x}\right)=4\sqrt{x}-3\sqrt{x^2}=\boxed{4\sqrt{x}-3x}$$

Don't forget to simplify any resulting roots that can be. That is an often missed part of these problems.

14. $\left(2\sqrt{x}+1\right)\left(3-4\sqrt{x}\right)$

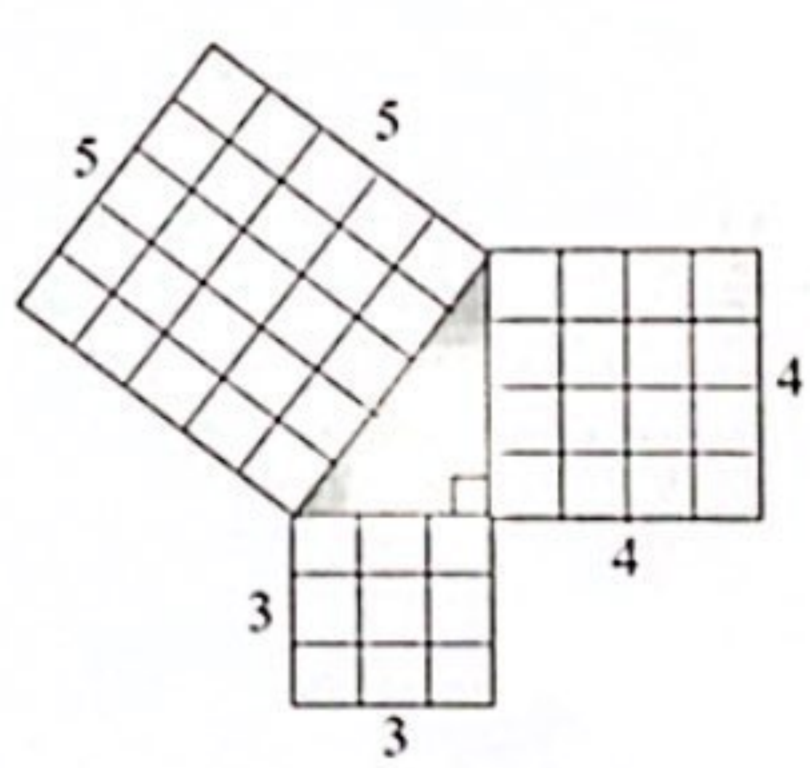
15. $\left(\sqrt[3]{x}+2\sqrt[3]{x^2}\right)\left(4-\sqrt[3]{x^2}\right)$

Pythagorean Theorem:



area A + area B = area C

$a^2 + b^2 = c^2$

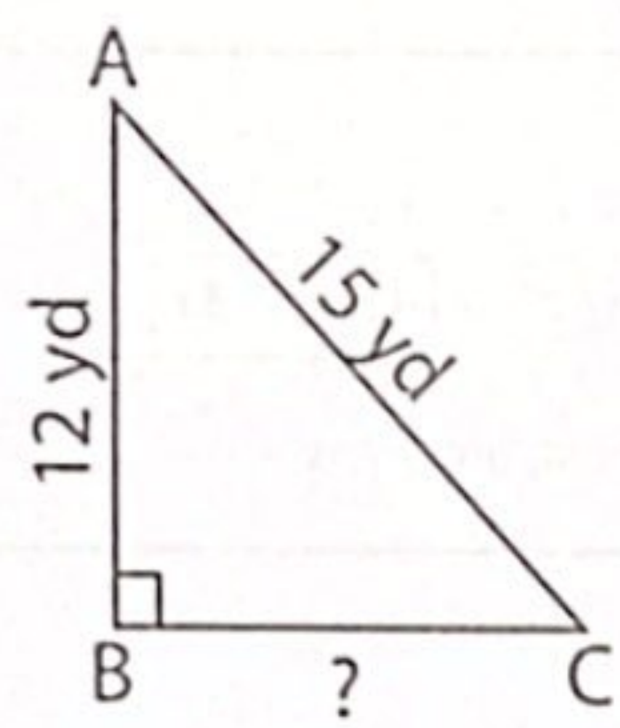


$3^2 + 4^2 = 5^2$

$9 + 16 = 25$

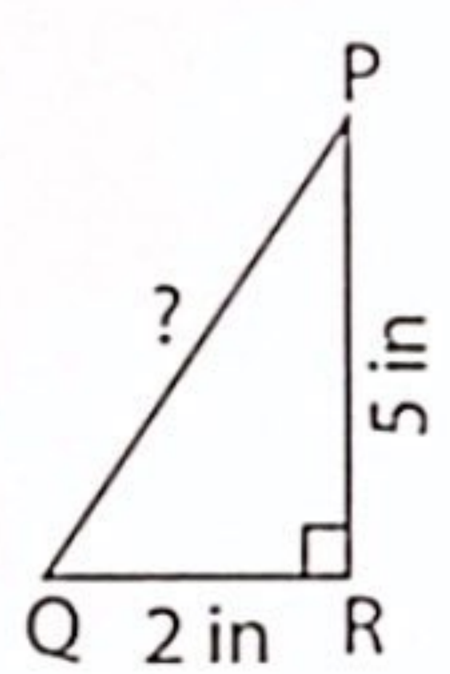
Find the missing length

16.



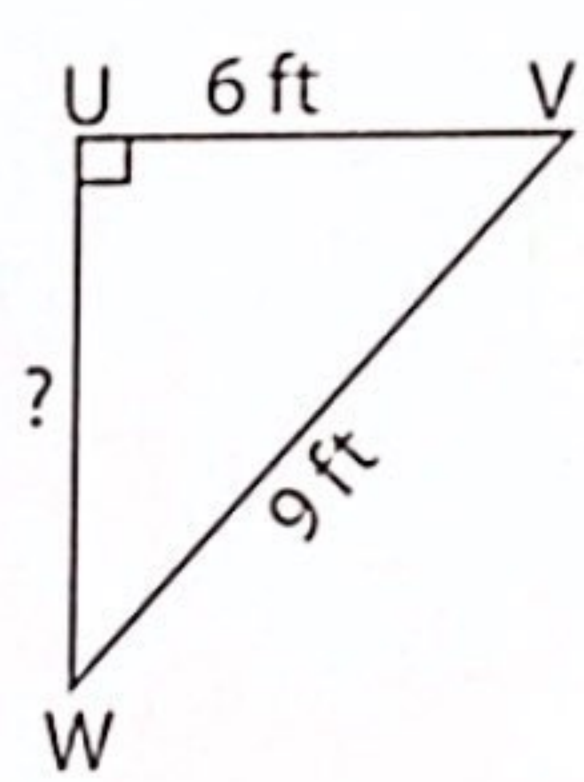
BC = ____________

17.



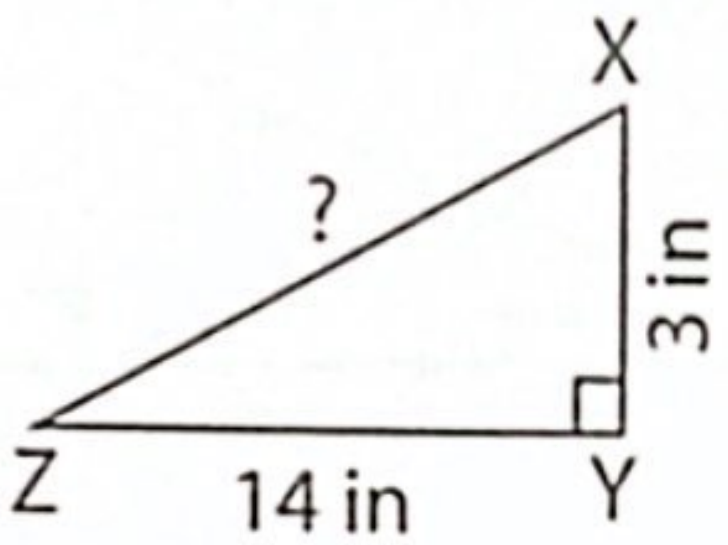
PQ = ____________

18.



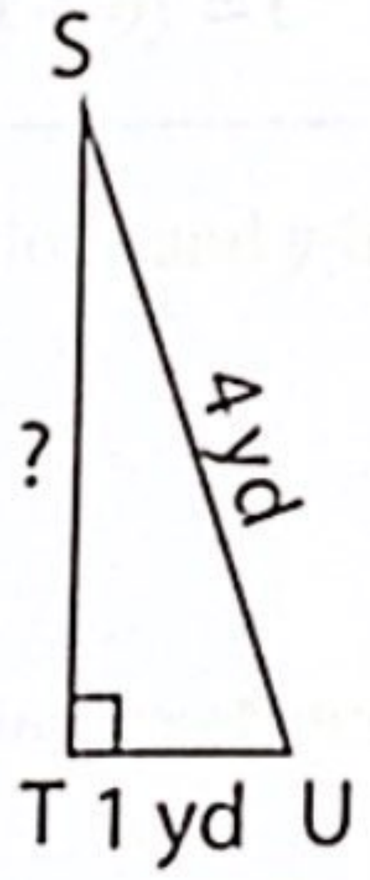
UW = ____________

19.



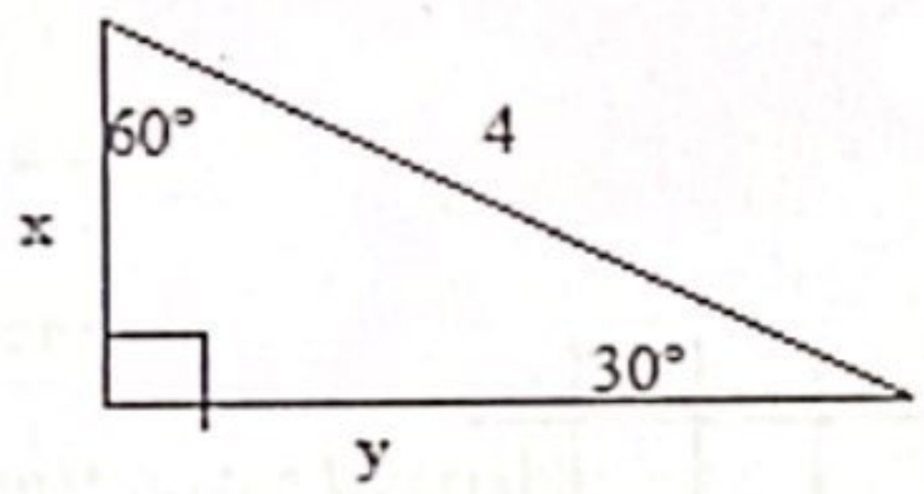
XZ = ____________

20.

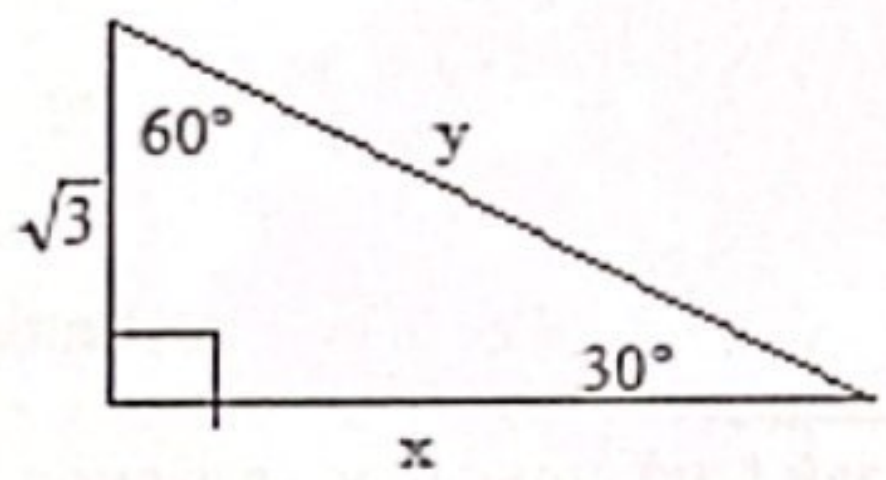


ST = ____________

21.



22.



Equations of Lines:	
Slope-intercept form: $y = mx + b$	Vertical line: $x = c$ *(slope is undefined)*
Point-slope form: $y - y_1 = m(x - x_1)$	Horizontal line: $y = c$ *(slope is zero)*
Standard Form: $Ax + By = C$	Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$

23. State the slope and y-intercept of the linear equation: $5x - 4y = 8$

24. Find the x-intercept and y-intercept of the equation: $2x - y = 5$

25. Write the equation in standard form: $y = 7x - 5$

Write the equation of the line in slope-intercept form with the following conditions:

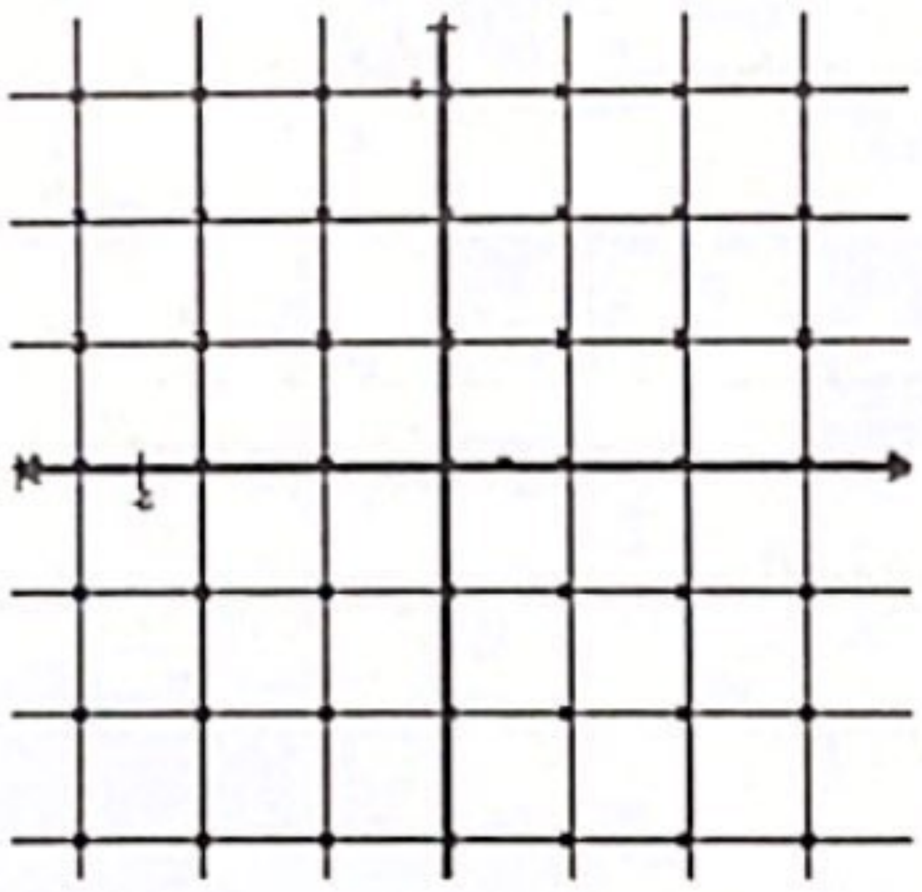
26. slope = -5 and passes through the point (-3, -8)

27. passes through the points (4, 3) and (7, -2)

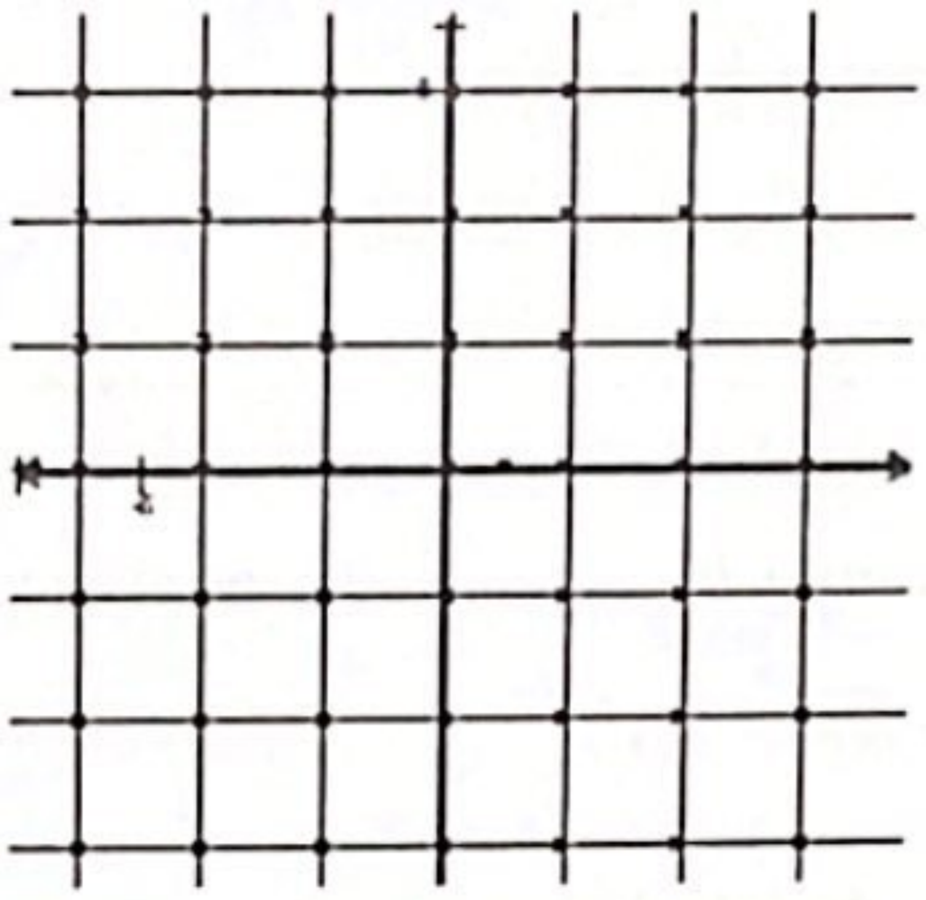
28. x-intercept = 3 and y-intercept =2

Graphing: Graph each function, inequality, and/or system.

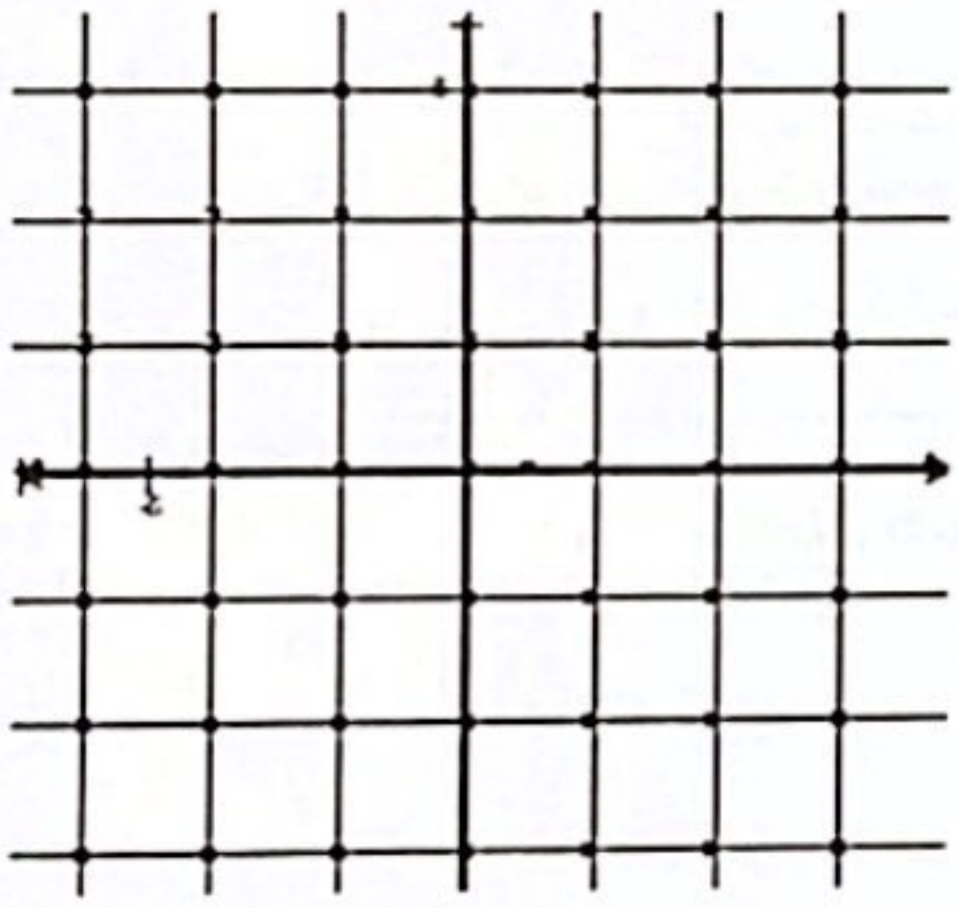
29. $3x-4y=12$



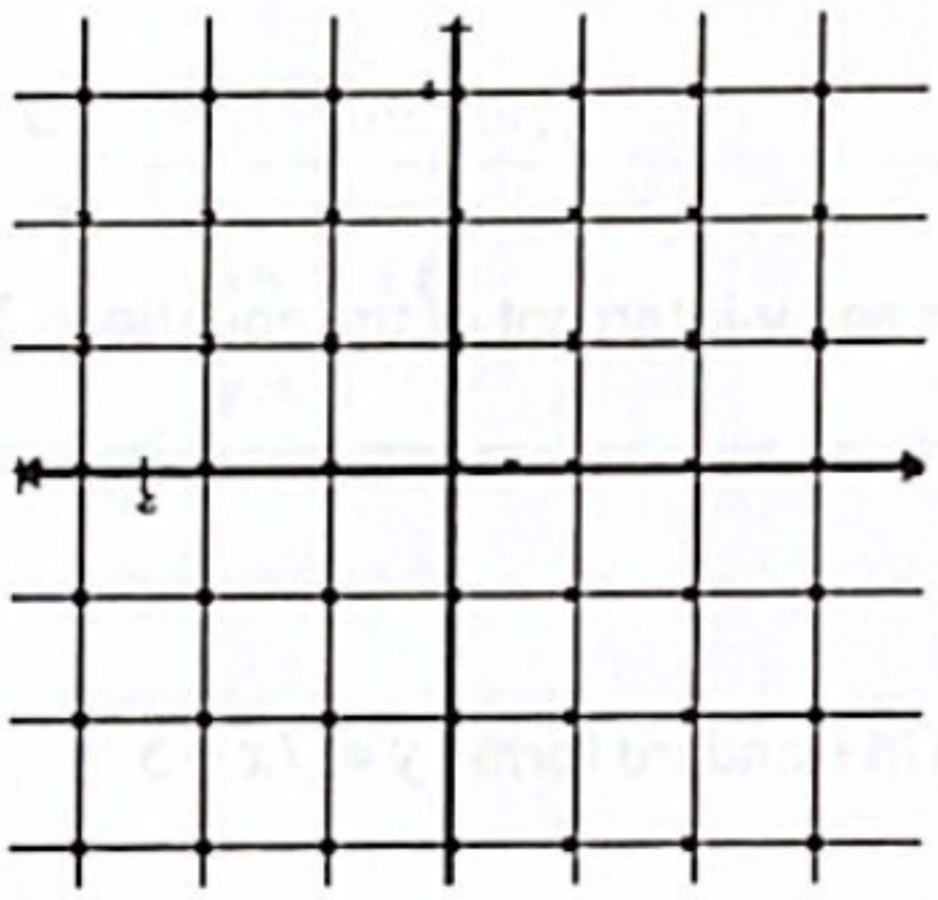
30. $\begin{cases} 2x+y=4 \\ x-y=2 \end{cases}$



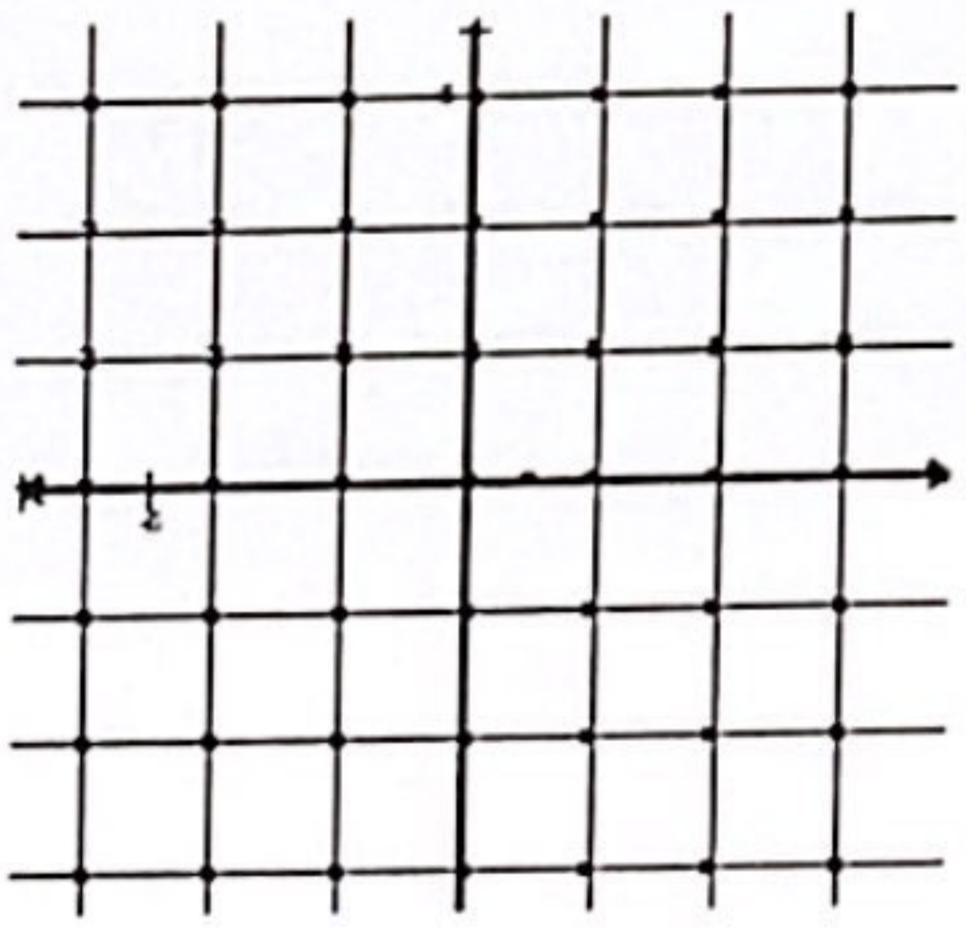
31. $y<-4x-2$



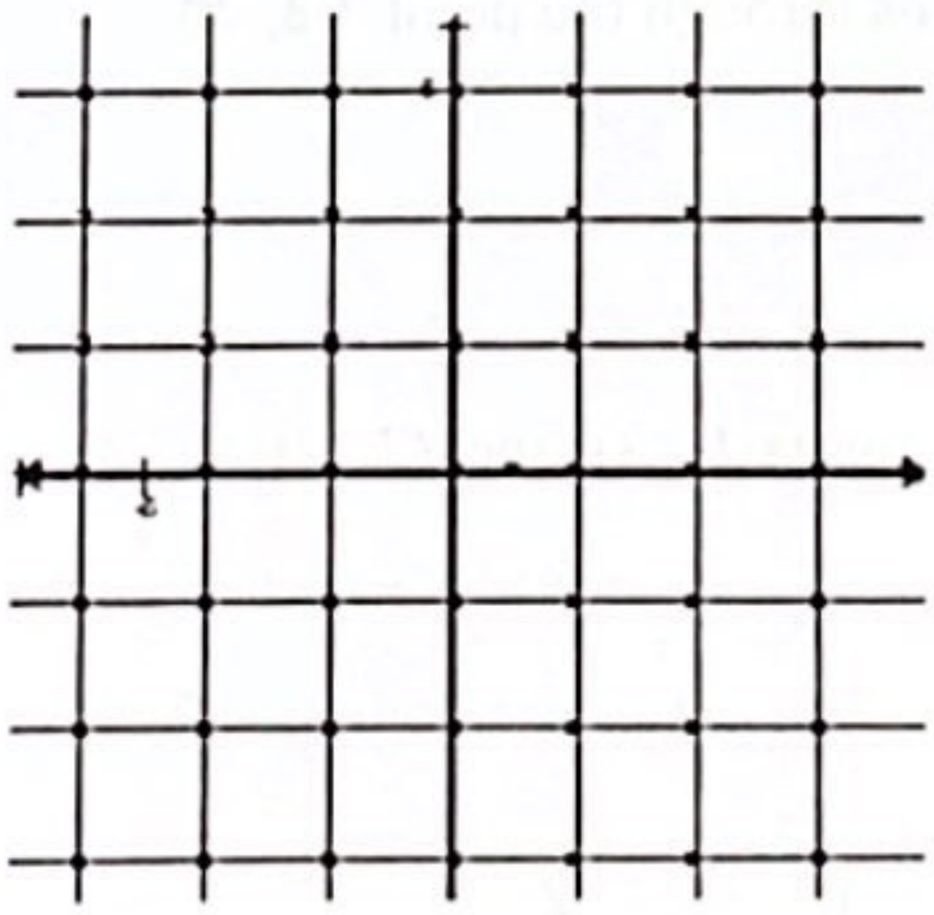
32. $y+2=|x+1|$



33. $y>|x|-1$



34. $y+4=(x-1)^2$



Systems of Equations:

$$\begin{cases} 3x + y = 6 \\ 2x - 2y = 4 \end{cases}$$

Substitution:		**Elimination:**	
Solve 1 equation for 1 variable		Find opposite coefficients for 1 variable	
Rearrange.		Multiply equation(s) by constant(s).	
Plug into 2nd equation.		Add equations together (lose 1 variable)	
Solve for the other variable.		Solve for variable.	

Then plug answer back into an original equation to solve for the 2nd variable.

$y = 6 - 3x$	Solve 1st equation for y	$6x + 2y = 12$	Multiply 1st equation by 2
$2x - 2(6 - 3x) = 4$	Plug into 2nd equation	$\underline{2x - 2y = 4}$	coefficients of y are opposite
$2x - 12 + 6x = 4$	Distribute	$8x = 16$	Add
$8x = 16$ and $x = 2$	Simplify	$x = 2$	Simplify.

Plug x=2 back into the original equation $6 + y = 6$, $y = 0$

Solve each system of equations, using any method.

35. $\begin{cases} 2x + y = 4 \\ 3x + 2y = 1 \end{cases}$

36. $\begin{cases} 2x + y = 4 \\ 3x - y = 14 \end{cases}$

37. $\begin{cases} 2w - 5z = 13 \\ 6w + 3z = 10 \end{cases}$

Exponents:

Recall the following rules of exponents:

1.	$a^1 = a$	Any number raised to the power of one equals itself.
2.	$1^a = 1$	One raised to any power is one.
3.	$a^0 = 1$	Any nonzero number raised to the power of zero is one.
4.	$a^m \cdot a^n = a^{m+n}$	When multiplying two powers that have the same base, add the exponents.
5.	$\frac{a^m}{a^n} = a^{m-n}$	When dividing two powers with the same base, subtract the exponents.
6.	$(a^m)^n = a^{mn}$	When a power is raised to another power, multiply the exponents.
7.	$a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$	Any nonzero number raised to a negative power equals its reciprocal raised to the opposite positive power.

Express each of the following in simplest form. Answers should not have any negative exponents.

38. $5a^0$

39. $\frac{3c}{c^{-1}}$

40. $\frac{2ef^{-1}}{e^{-1}}$

41. $\frac{\left(n^3p^{-1}\right)^2}{(np)^{-2}}$

Simplify.

42. $3m^2 \bullet 2m$

43. $\left(a^3\right)^2$

44. $(-b^3c^4)^5$

45. $4m(3a^2m)$

Polynomials:

To add/subtract polynomials, combine like terms.

EX: $8x-3y+6-(6y+4x-9)$ *Distribute the negative through the parantheses.*

$=8x-3y+6-6y-4x+9$ *Combine like terms with similar variables.*

$=8x-4x-3y-6y+6+9$

$=4x-9y+15$

Simplify.

46. $3x^3+9+7x^2-x^3$

47. $7m-6-(2m+5)$

To multiply two binomials, use FOIL.

EX: $(3x-2)(x+4)$ *Multiply the first, outer, inner, and last terms.*

$=3x^2+12x-2x-8$ *Combine like terms together.*

$=3x^2+10x-8$

Multiply.

48. $(3a+1)(a-2)$

49. $(s+3)(s-3)$

50. $(c-5)^2$

51. $(5x+7y)(5x-7y)$

Factor completely.

52. $z^2 + 4z - 12$

53. $6 - 5x - x^2$

54. $2k^2 + 2k - 60$

55. $-10b^4 - 15b^2$

56. $9c^2 + 30c + 25$

57. $9n^2 - 4$

58. $27z^3 - 8$

59. $2mn - 2mt + 2sn - 2st$

To solve quadratic equations, try to factor first and set each factor equal to zero. Solve for your variable. If the quadratic does NOT factor, use the quadratic formula.

EX:	$x^2 - 4x = 21$	*Set equal to zero FIRST.*
	$x^2 - 4x - 21 = 0$	*Now factor.*
	$(x+3)(x-7) = 0$	*Set each factor equal to zero.*
	$x + 3 = 0 \quad x - 7 = 0$	*Solve for each x.*
	$x = -3 \quad x = 7$	

Solve each equation.

60. $x^2 - 4x - 12 = 0$

61. $x^2 + 25 = 10x$

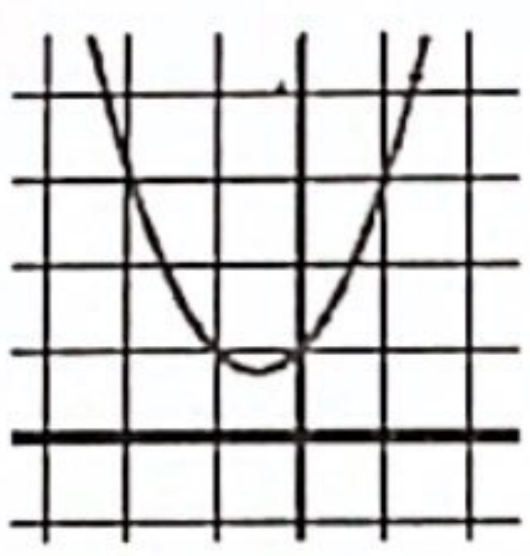
62. $x^2 - 14x + 40 = 0$

Discriminant: The number under the radical in the quadratic formula (b^2-4ac) can tell you what kind of roots you will have.

If $b^2-4ac>0$ you will have TWO real roots (touches the x-axis twice)

If $b^2-4ac=0$ you will have ONE real root (touches axis once)

If $b^2-4ac<0$ you will have TWO imaginary roots. (Function does not cross the x-axis)



QUADRATIC FORMULA—allows you to solve any quadratic for all its real and imaginary roots.

$5x^2-2x+4=0$ $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

EX: In the equation $x^2+2x+3=0$, find the value of the discriminant, describe the nature of the roots, then solve.

$x^2+2x+3=0$ *Determine the values of a, b, and c.*

$a=1$ $b=2$ $c=3$ *Find the discriminant.*

$D=2^2-4\cdot1\cdot3$

$D=4-12$

$D=-8$ *There are two imaginary roots.*

$Solve:\ x=\dfrac{-2\pm\sqrt{-8}}{2}$

$x=\dfrac{-2\pm2i\sqrt{2}}{2}$

$x=-1\pm i\sqrt{2}$

Find the value of the discriminant, describe the nature of the roots, then solve each quadratic. Use EXACT values.

63. $x^2 - 9x + 14 = 0$

Discriminant = ____________________

Type of Roots: ____________________

Exact Value of Roots: ________________

64. $5x^2 - 2x + 4 = 0$

Discriminant = ____________________

Type of Roots: ____________________

Exact Value of Roots: ________________

Long Division—can be used when dividing any polynomials.

Evaluate each function for the given value.

67. $f(x)=x^2-6x+2$

$f(3)=$________

68. $g(x)=6x-7$

$g(x+h)=$________

69. $f(x)=3x^2-4$

$5[f(x+2)]=$________

Composition and Inverses of Functions:

Recall: $(f \circ g)(x)=f(g(x))$ OR $f[g(x)]$ read "**f of g of x**" means to plug the inside function in for x in the outside function.

Example: Given $f(x)=2x^2+1$ *and* $g(x)=x-4$ find $f(g(x))$.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2+1 \\ &= 2(x^2-8x+16)+1 \\ &= 2x^2-16x+32+1 \\ f(g(x)) &= 2x^2-16x+33 \end{aligned}$$

Suppose $f(x)=2x,\ g(x)=3x-2,$ *and* $h(x)=x^2-4$. **Find the following:**

70. $f[g(2)]=$________

71. $f[g(x)]=$________

72. $f[h(3)]=$________

73. $g[f(x)]=$________

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.

Multiplying and Dividing: **Factor numerator and denominator completely. Cancel any common factors in the top and bottom. If dividing, change divide to multiply and flip the second fraction.**

EX: $\frac{x^2+10x+21}{5-4x-x^2} \bullet \frac{x^2+2x-15}{x^3+4x^2-21x}$ *Factor everything completely.*

$= \frac{(x+7)(x+3)}{(5+x)(1-x)} \bullet \frac{(x+5)(x-3)}{x(x-3)(x+7)}$ *Cancel out common factors in the top and bottom.*

$= \frac{(x+3)}{x(1-x)}$ *Simplify.*

76. $\frac{5z^3+z^2-z}{3z}$

77. $\frac{m^2-25}{m^2+5m}$

78. $\frac{10r^5}{21s^2} \bullet \frac{3s}{5r^3}$

79. $\frac{a^2-5a+6}{a+4} \bullet \frac{3a+12}{a-2}$

80. $\frac{6d-9}{5d+1} \div \frac{6-13d+6d^2}{15d^2-7d-2}$

Addition and Subtraction

First find the least common denominator. Write each fraction with that LCD. Add/subtract numerators as indicated and leave the denominators as they are.

EX:	$\frac{3x+1}{x^2+2x}+\frac{5x-4}{2x+4}$	*Factor denominator completely.*
	$\frac{3x+1}{x(x+2)}+\frac{5x-4}{2(x+2)}$	*Find LCD, which is* $(2x)(x+2)$
	$\frac{2(3x+1)}{2x(x+2)}+\frac{x(5x-4)}{2x(x+2)}$	*Rewrite each fraction with the LCD in the denominator.*
	$\frac{6x+2+5x^2-4x}{2x(x+2)}$	*Write as one fraction.*
	$\frac{5x^2+2x+2}{2x(x+2)}$	*Combine like terms.*

81. $\frac{2x}{5}-\frac{x}{3}$

82. $\frac{b-a}{a^2b}+\frac{a+b}{ab^2}$

83. $\frac{2-a^2}{a^2+a}+\frac{3a+4}{3a+3}$

Complex Fractions: Eliminate complex fractions by multiplying the numerator and denominator by the LCD of each of the small fractions. Then simplify the result.

EX: $\dfrac{1+\frac{1}{a}}{\frac{2}{a^2}-1}$ *Find LCD: a^2*

$=\dfrac{\left(1+\frac{1}{a}\right)\bullet a^2}{\left(\frac{2}{a^2}-1\right)\bullet a^2}$ *Multiply top and bottom by LCD.*

$=\dfrac{a^2+a}{2-a^2}$ *Factor and simplify if possible.*

$=\dfrac{a(a+1)}{2-a^2}$

84. $\dfrac{1-\frac{1}{2}}{2+\frac{1}{4}}$

85. $\dfrac{1+\frac{1}{z}}{z+1}$

86. $\dfrac{5+\frac{1}{m}-\frac{6}{m^2}}{\frac{2}{m}-\frac{2}{m^2}}$

87. $\dfrac{2+\frac{1}{x}-\frac{1}{x^2}}{1+\frac{4}{x}+\frac{3}{x^2}}$

Solving Rational Equations:

Multiply each term by the LCD of all the fractions. This should eliminate all of our fractions. Then solve the equation as usual.

$$\frac{5}{x+2}+\frac{1}{x}=\frac{5}{x}$$ *Find LCD first* $x(x+2)$

$$x(x+2)\frac{5}{x+2}+x(x+2)\frac{1}{x}=\frac{5}{x}x(x+2)$$ *Multiply each term by the LCD.*

$$5x+1(x+2)=5(x+2)$$ *Simplify and solve.*

$$5x+x+2=5x+10$$

$$6x+2=5x+10$$

$x=8$ ← *Check your answer! Sometimes they do not check!*

Check:

$$\frac{5}{8+2}+\frac{1}{8}=\frac{5}{8}$$

$$\frac{5}{10}+\frac{1}{8}=\frac{5}{8}$$

$$\frac{5}{8}=\frac{5}{8}$$

Solve each equation. Check your solutions.

88. $\frac{12}{x}+\frac{3}{4}=\frac{3}{2}$

89. $\frac{x+10}{x^2-2}=\frac{4}{x}$

90. $\frac{5}{x-5}=\frac{x}{x-5}-1$